APPLICATION OF APPROXIMATE METHODS TO THE PROBLEM OF HEATING OF MASSIVE BODIES AT A CONSTANT AND VARIABLE WATER EQUIVALENT OF GASES

M. K. Kleiner

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Approximate solutions of problems of thermal conductivity in a moving layer are obtained by the Brovman–Surin method [5] and integral heat balance method [8].

Solutions of the equation of thermal conductivity for the case of heating of bodies in a parallel flow and in a counterflow have a complex form even in the case of a constant water equivalent of gases [1-4]. In addition, calculation of the eigenvalues of the problem is a quite laborious operation which complicates the application of the solutions of linear problems to calculation of processes with variable characteristics by means of successive approximations. Approximate solutions of linear problems are given below.

1. The Water Equivalent of Gases is Constant. The problem is formulated as follows:

$$\frac{\partial t(\rho,\ X)}{\partial X}=\frac{\partial^2 t(\rho,\ X)}{\partial\rho^2}+\frac{2\nu+1}{\rho}\frac{\partial t(\rho,\ X)}{\partial\rho},\tag{1}$$

$$-\frac{\partial t(1,\ X)}{\partial\rho}+\mathrm{Bi}\,[t_g(X)-t(1,\ X)]=0,\quad \frac{\partial t(0,\ X)}{\partial\rho}=0,\tag{2}$$

$$\frac{m_0}{2\nu+2}\frac{dt_g(X)}{dX}=-\frac{m_0 t_{\max}}{2\nu+2}\frac{dq_3^{\mathrm{av}}(X)}{dX}\mp\mathrm{Bi}\,[t_g(X)-t(1,\ X)]\mp\mathrm{Bi}_1\,[t_g(X)-t_{\mathrm{en}}],\tag{3}$$

$$t(\rho,\ 0)=t_0(\rho),\ t_g(0)=t_{gi}\ \ t(0,\ X)<\infty.\tag{4}$$

The upper sign is for the parallel flow and the lower for the counterflow.

$$\rho=\frac{r}{R},\ X=\frac{ax}{vR^2},\ \mathrm{Bi}=\frac{\alpha R}{\lambda},\ \mathrm{Bi}_1=\frac{KR}{\lambda}\frac{\Pi}{f},$$

$$m_0=\frac{w_g}{Gc},\ w_g=v_g c_g,\ t_{\max}=\frac{Q_n^r}{L_g c_g},\ v=\frac{2\nu+2}{\gamma f R}G,$$

$$f_{\mathrm{pl}}=2b,\ f_{\mathrm{cy}}=\frac{\pi b}{s_1/2R},\ f_{\mathrm{sp}}=\frac{3}{R}F(1-f_{01}).$$

According to [5], we seek the approximate solution of problem (1)-(4) in the form

$$t(\rho,\ X)=\sum_{i=0}^{4}f_i(X)\rho^i.\tag{5}$$

To fulfill the second of conditions (2) $f_1=f_3\equiv 0$. Substituting (5) into (1), equating terms with the same powers of ρ, and neglecting the quantity $\rho^4 f_4'(X)$ in comparison with the others, we obtain the relations*

*Here and henceforth differentiation with respect to X is denoted by a prime.

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$$f_2 = \frac{f_0'}{2(2\nu+2)}, \quad f_4 = \frac{f_0''}{2\cdot 4(2\nu+2)(2\nu+4)}. \tag{6}$$

Then substituting (5) with consideration of (6) into (2) and (3) and solving them simultaneously, we obtain the following differential equation for $f_0(X)$:

$$d_3 f_0''' + d_2 f_0'' + d_1 f_0' + d_0 f_0 = d_{00}(X) \pm \mathrm{Bi}_1 t_{en}$$

$$d_3 = \frac{m_0}{2\nu+2} \frac{1}{2(2\nu+2)(2\nu+4)} \left(\frac{1}{4} + \frac{1}{\mathrm{Bi}}\right), \quad d_0 = \pm \mathrm{Bi}_i, \tag{7}$$

$$d_2 = \frac{1}{2\nu+2} \left\{\frac{m_0}{2\nu+2}\left(\frac{1}{2} + \frac{1}{\mathrm{Bi}}\right) \pm \frac{1}{2(2\nu+4)}\left[1 + \mathrm{Bi}_1\left(\frac{1}{4} + \frac{1}{\mathrm{Bi}}\right)\right]\right\},$$

$$d_1 = \frac{m_0}{2\nu+2} \pm \frac{1}{2\nu+2}\left[1 + \mathrm{Bi}_1\left(\frac{1}{2} + \frac{1}{\mathrm{Bi}}\right)\right], \quad d_{00}(X) = \frac{m_0 t_{\max}}{2\nu+2} \frac{dq_3^{av}(X)}{dX}. \tag{8}$$

Having determined f_0 from (7) and substituting it into (5), with consideration of (6) we obtain the following approximate expression for $t(\rho, X)$:

$$t(\rho, X) = t_{en} + D(X) + \frac{\rho^2 D'(X)}{2(2\nu+2)} + \frac{\rho^4 D''(X)}{2\cdot 4(2\nu+2)(2\nu+4)}$$

$$+ \sum_{i=1}^{3} C_i \left[1 + \frac{\rho^2 \delta_i}{2(2\nu+2)} + \frac{\rho^4 \delta_i^2}{2\cdot 4(2\nu+2)(2\nu+4)}\right] \exp(\delta_i X). \tag{9}$$

Here D(X) is the particular solution of inhomogeneous equation (7) with right-hand side $d_{00}(X)$; the δ_i are the roots of the characteristic equation

$$d_3\delta^3 + d_2\delta^2 + d_1\delta + d_0 = 0. \tag{10}$$

We obtain the expression for the temperature of the gases from boundary condition (2) by means of (9)

$$t_g(X) = t_{en} + D(X) + \frac{D'(X)}{2\nu+2}\left(\frac{1}{2} + \frac{1}{\mathrm{Bi}}\right)$$

$$+ \frac{D''(X)}{2(2\nu+2)(2\nu+4)}\left(\frac{1}{4} + \frac{1}{\mathrm{Bi}}\right) + \sum_{i=1}^{3} C_i \left[1 + \frac{\delta_i}{2\nu+2}\left(\frac{1}{2} + \frac{1}{\mathrm{Bi}}\right)\right.$$

$$\left. + \frac{\delta_i^2}{2(2\nu+2)(2\nu+4)}\left(\frac{1}{4} + \frac{1}{\mathrm{Bi}}\right)\right] \exp(\delta_i X). \tag{11}$$

An analysis shows that for real cases roots (10) are prime, real numbers, all of them being negative for the case of parallel flow and one of them being positive for counterflow.

If the law of burning of fuel can be approximated by the exponential curve

$$q_3^{av}(X) = q_3^0 \exp(-kX) \quad \text{for parallel flow,}$$

$$q_3^{av}(X) = q_3^0 \exp[-k(L_1 - X)] \quad \text{for counterflow,}$$

then D(X) has the form

$$D(X) = -\frac{m_0 k t_{\max} q_3^{av}(X)}{(2\nu+2)(d_3 k^3 \mp d_2 k^2 + d_1 k \mp d_0)}. \tag{12}$$

The average temperature of the body is

$$t_{av}(X) = t_{en} + D(X) + \frac{D'(X)}{2(2\nu+4)} + \frac{D''(X)}{2\cdot 4(2\nu+4)(2\nu+6)}$$

$$+ \sum_{i=1}^{3} C_i \left[1 + \frac{\delta_i}{2(2\nu+4)} + \frac{\delta_i^2}{2\cdot 4(2\nu+4)(2\nu+6)}\right] \exp(\delta_i X). \tag{13}$$

The coefficients C_i can be determined by the well-known method [6] from system of Eqs. (9), (11), and (13) so that some of the initial conditions

$$t_g(0) = t_{gi}, \quad t(1,\ 0) - t(0,\ 0) = \Delta t_m^0,$$

$$t_{av}(0) = t_{av}^0 = (2\nu + 2) \int_0^1 t_0(\rho)\, \rho^{2\nu+1} d\rho$$

are fulfilled for X = 0.

2. The Water Equivalent of the Gases is Variable. In this case it is expedient to take a positive direction with respect to the path of the gases both for the parallel flow and counterflow. We will assume that the water equivalent of the gases in the direction of their travel increases linearly:

$$w_g = a_0 + a_1 z, \tag{14}$$

i.e., the fuel is delivered through end burners and uniformly through side burners. Then the problem reduces to solving the system:

$$\pm \frac{\partial t(\rho,\ Z)}{\partial Z} = \frac{\partial^2 t(\rho,\ Z)}{\partial \rho^2} + \frac{2\nu+1}{\rho} \frac{\partial t(\rho,\ Z)}{\partial \rho}, \tag{15}$$

$$-\frac{\partial t(1,\ Z)}{\partial \rho} + \mathrm{Bi}\,[t_g(Z) - t(1,\ Z)] = 0, \quad \frac{\partial t(0,\ Z)}{\partial \rho} = 0, \tag{16}$$

$$\left(\frac{m_{01}}{2\nu+2} + m_1 \mathrm{Bi}\, Z\right) \frac{dt_g(Z)}{dZ} = -\frac{R}{\lambda f} \frac{dQ}{dz} + m_1 \mathrm{Bi}\,[t_g^0 - t_g(Z)] - \mathrm{Bi}\,[t_g(Z) - t(1,\ Z)] - \mathrm{Bi}_1\,[t_g(Z) - t_{en}], \tag{17}$$

$$t(\rho,\ 0) = t_0(\rho), \quad t_g(0) = t_{gi},$$

$$Z = \frac{az}{vR^2}, \quad m_{01} = \frac{a_0}{Gc}, \quad m_1 = \frac{a_1}{\alpha f}. \tag{18}$$

The upper sign is for parallel flow and the lower for counterflow. The solution of a similar problem for thin bodies when Q = const is given in [7].

The solution of system (15)-(18) is found by means of the integral heat balance method [8] in the form

$$t(\rho,\ Z) = A_0(Z) + A_1(Z)\, \rho^n. \tag{19}$$

Expression (19) satisfies the symmetry condition when $n > 1$.

Substituting (19) into (15) and integrating the left- and right-hand sides with respect to ρ from zero to unity, we obtain

$$\pm \frac{dA_0(Z)}{dZ} \pm \frac{1}{n+1} \frac{dA_1(Z)}{dZ} = \frac{n}{n-1} (2\nu + n)\, A_1(Z). \tag{20}$$

Solving simultaneously (16) and (17) with consideration of (19) and then applying the Laplace transform to the numerical expression and to Eq. (20), we obtain the following system of equations:

$$\bar{A}_0 = \pm \frac{n}{n-1} \frac{2\nu+n}{s} \bar{A}_1 + \frac{1}{s}\left(A_0^0 + \frac{A_1^0}{n+1}\right) - \frac{\bar{A}_1}{n+1}, \tag{21}$$

$$m_1 \mathrm{Bi}\, s \frac{d}{ds}\left[\bar{A}_0 + \left(1 + \frac{n}{\mathrm{Bi}}\right)\bar{A}_1\right] - \bar{A}_0\left(\frac{m_{01}}{2\nu+2} s + \mathrm{Bi}_1\right)$$

$$-\bar{A}_1\left[\frac{m_{01}}{2\nu+2} s\left(1 + \frac{n}{\mathrm{Bi}}\right) + n + \mathrm{Bi}_1\left(1 + \frac{n}{\mathrm{Bi}}\right)\right] = \Phi(s) - \frac{m_{01}}{2\nu+2} t_{gi} - \frac{m_1 \mathrm{Bi}\, t_g^0 + \mathrm{Bi}_1 t_{en}}{s}. \tag{22}$$

In (21)-(22) the superior bar denotes Laplace-transformed functions $A_0(Z)$ and $A_1(Z)$, it being assumed that $A_0(0) = A_0^0$, $A_1(0) = A_1^0$,

$$\Phi(s) = \frac{R}{\lambda f} \int_0^\infty \frac{dQ}{dz} \exp(-sZ)\, dZ.$$

Solving simultaneously (21) and (22) and assuming that $\Phi(s)$ can be represented as a series*

*This holds true if Q(z) is expanded in powers of z.

$$\Phi(s)=\sum_{m=1}^{\infty}\frac{l_m}{s^m}, \tag{23}$$

we obtain for $\overline{A}_1$ the following equation:

$$\frac{d\overline{A}_1}{ds}(g_0+g_1s)+\overline{A}_1\left(\frac{b_{-1}}{s}+b_0+b_1s\right)+\frac{c_{-1}}{s}+c_0-\sum_{m=1}^{\infty}\frac{l_m}{s^m}=0, \tag{24}$$

$$\begin{gathered}
g_0=\pm m_1\mathrm{Bi}\frac{n}{n-1}(2\nu+n),\quad g_1=m_1\frac{n(\mathrm{Bi}+n+1)}{n+1},\\
b_{-1}=\mp(m_1\mathrm{Bi}+\mathrm{Bi}_1)\frac{n}{n-1}(2\nu+n),\\
b_0=\mp\frac{m_{01}}{2\nu+2}\frac{n}{n-1}(2\nu+n)-\mathrm{Bi}_1\frac{n(\mathrm{Bi}+n+1)}{\mathrm{Bi}(n+1)}-n,\\
b_1=-\frac{m_{01}}{2\nu+2}\frac{n(\mathrm{Bi}+n+1)}{\mathrm{Bi}(n+1)},\\
c_{-1}=(m_1\mathrm{Bi}\,t_g^0+\mathrm{Bi}_1t_{en})-(m_1\mathrm{Bi}+\mathrm{Bi}_1)\left(A_0^0+\frac{A_1^0}{n+1}\right),\\
c_0=\frac{m_{01}}{2\nu+2}\left[t_{gi}-\left(A_0^0+\frac{A_1^0}{n+1}\right)\right].
\end{gathered} \tag{25}$$

We seek the solution of (24) in the form

$$\overline{A}_1=\sum_{m=1}^{\infty}\frac{D_m}{s^m}. \tag{26}$$

Substituting (26) into (24) and equating the coefficients of like powers of s to zero, we obtain the following recursion system for determining D_m:

$$\begin{gathered}
b_1D_1+c_0=0,\\
(b_0-g_1)D_1+c_{-1}+b_1D_2-l_1=0,\\
(b_{-1}-g_0m)D_m+[b_0-g_1(m+1)]D_{m+1}+b_1D_{m+2}-l_{m+1}=0.
\end{gathered} \tag{27}$$

From the first relation we have

$$D_1=\frac{\mathrm{Bi}(n+1)}{n(\mathrm{Bi}+n+1)}\left[t_{gi}-\left(A_0^0+\frac{1}{n+1}A_1^0\right)\right].$$

Knowing $\overline{A}_1$, we can determine $\overline{A}_0$ from (21). Having performed an inverse Laplace transformation, we finally find the expression for $t(\rho, Z)$

$$t(\rho,Z)=A_0^0+\frac{A_1^0}{n+1}+D_1\left(\rho^n-\frac{1}{n+1}\right)$$
$$+\sum_{m=1}^{\infty}\frac{Z^m}{m!}\left[\pm\frac{n}{n-1}(2\nu+n)D_m+\left(\rho^n-\frac{1}{n+1}\right)D_{m+1}\right]. \tag{28}$$

We can determine the temperature of the gases by using (16) and (28):

$$t_g(Z)=A_0^0+\frac{1}{n+1}A_1^0+D_1\frac{n(\mathrm{Bi}+n+1)}{\mathrm{Bi}(n+1)}$$
$$+\sum_{m=1}^{\infty}\frac{Z^m}{m!}\left[\pm\frac{n}{n-1}(2\nu+n)D_m+\frac{n(\mathrm{Bi}+n+1)}{\mathrm{Bi}(n+1)}D_{m+1}\right]. \tag{29}$$

The average temperature of the body is

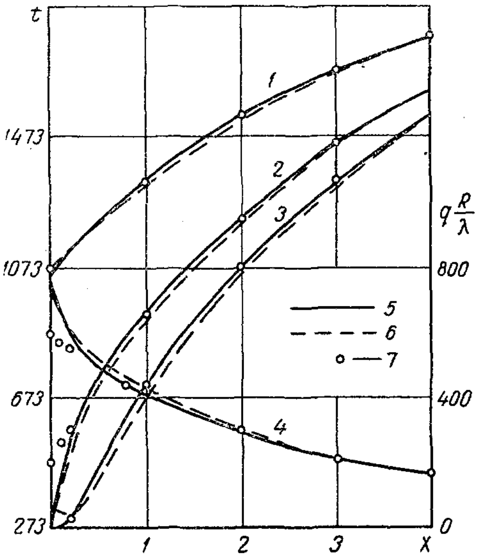

Fig. 1

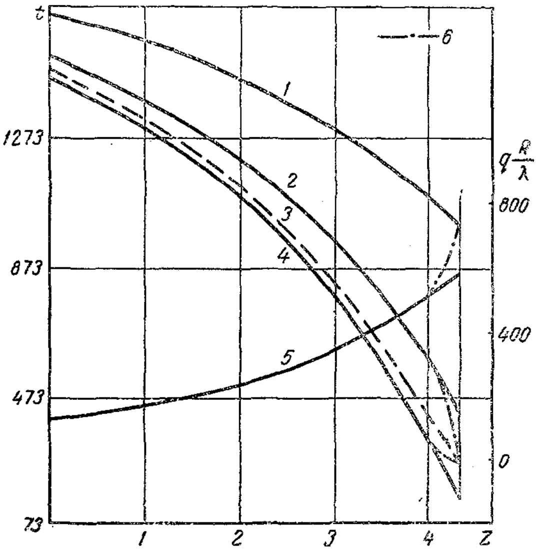

Fig. 2

Fig. 1. Heating of a plate in a counterflow for Bi = 1, Bi_1 = 0.02, m_0 = 2, $q_3^{av}(X)$ = 0. 1) Temperature of gases; 2) of surface; 3) of axis; 4) q(R/λ); 5) exact solution; 6) approximate solution by the Brovman–Surin method (formulas (9), (11)); 7) the same, by integral heat balance method (formulas (28), (29)) for n = 2.

Fig. 2. Calculation of the heating of a plate in a counterflow for m_{01} = 0, m_1 = 0.25, Bi = 1, Bi_1 = 0.02, n = 2, Q(z) = const by formulas (28), (29). 1) Temperature of gases; 2) of surface; 3) average for mass; 4) of axis; 5) q(R/λ); 6) presumed change of corresponding values during initial period of heating.

$$t_{av}(Z) = A_0^0 + \frac{1}{n+1} A_1^0 + D_1 \frac{n(2\nu+1)}{(n+1)(2\nu+n+2)}$$

$$+ \sum_{m=1}^{\infty} \frac{Z^m}{m!} \left[\pm \frac{n}{n-1}(2\nu+n) D_m + \frac{n(2\nu+1)}{(n+1)(2\nu+n+2)} D_{m+1} \right]. \tag{30}$$

An investigation of the convergence of the series in (28)-(30) by means of the d'Alembert test [6] showed that when m_{01} = 0 they converge for all finite values of Z.

If $m_{01} \equiv 0$, it is no longer possible to assign the value of t_{gi}, since the water equivalent of the gases at point Z = 0 is also equal to zero. In this case we mean by t_{gi} its limiting value when $m_{01} \to 0$. It is determined by the second of relations (27) when $b_1 \to 0$.

We will assume that on the whole the flow of gases does not have an effect on the burning of fuel in an elementary jet. Then we can show that the quantity of chemical energy of the combustion products passing through a given section of the furnace in unit time is determined by the expression

$$Q(z) = a_0 t_{max} q_{3e}^{av}(z) + a_1 t_{max} \int_0^z q_{3s}^{av}(z-\varepsilon)\, d\varepsilon. \tag{31}$$

The subscripts e and s of $q_3^{av}(z)$ indicate that they pertain to the end and side burners.

Approximating $q_3^{av}(z)$ by the exponential curve

$$q_{3e}^{av}(z) = q_{3e}^0 \exp(-k_e z), \quad q_{3s}^{av}(z) = q_{3s}^0 \exp(-k_s z), \tag{32}$$

with consideration of (31) we obtain

$$\frac{R}{\lambda f} \frac{dQ}{dz} = -t_{max} \left[\frac{m_{01}}{2\nu+2} K_e q_{3e}^0 \exp(-K_e Z) - m_1 \,\mathrm{Bi}\, q_{3s}^0 \exp(-K_s Z) \right]. \tag{33}$$

TABLE 1. Comparison of the Approximate μ- and Exact $\tilde{\mu}$-Values of the Eigenvalues for Bi = 1.0, Bi_1 = 0.02, and Constant Ratio of Water Equivalents

m_0	Forward flow		Counterflow	
	μ	$\tilde{\mu}$	μ	$\tilde{\mu}$
0,5	0,115 1,47 3,47	0,115 1,48 2,58	0,922*i* 0,190 3,39	0,917*i* 0,188 2,53
2,0	0,0817 1,05 3,44	0,0816 1,05 2,55	0,14*i* 0,617 3,42	0,14*i* 0,617 2,54

Expanding the exponents of (33) in series and performing a direct Laplace transformation, we obtain the following expression for the coefficients l_m of expression (23):

$$l_m = (-1)^m t_{\max} \left(\frac{m_{01}}{2\nu + 2} K_e^m q_{3e}^0 - m_1 \mathrm{Bi}\, q_{3s}^0 K_s^{m-1} \right). \quad (34)$$

For check calculations, when the temperature of the gases at the site of charging the metal is known also for the counterflow, it is advantageous to place the origin of the coordinates at the site of charging the metal. In this case expressions (27)-(30) hold true; in (25), as is usual for a counterflow, it is necessary to take the lower signs everywhere and only thereafter take the values of g_1, b_0, and c_{-1} with the opposite sign, i.e.,

$$g_1 = -m_1 \frac{n(\mathrm{Bi} + n + 1)}{n+1},$$
$$b_0 = -\frac{m_{01}}{2\nu + 2} \frac{n}{n-1} (2\nu + n) + \mathrm{Bi}_1 \frac{n(\mathrm{Bi} + n + 1)}{\mathrm{Bi}(n+1)} + n, \quad (25a)$$
$$c_{-1} = -(m_1 \mathrm{Bi}\, t_g^0 + \mathrm{Bi}_1 t_{en}) + (m_1 \mathrm{Bi} + \mathrm{Bi}_1)\left(A_0^0 + \frac{A_1^0}{n+1} \right).$$

Here it is necessary to bear in mind that the value of m_{01} is taken with respect to the water equivalent of the gases at the site of charging the metal. In addition, in (31) and (32) it is necessary to replace z by L – z. Accordingly (33)-(34) will also change.

An analysis of expressions (28)-(30) obtained shows that in the general case the temperature distribution at the initial instant does not correspond to that given $[t(0, 0) \neq A_0^0,\ t(1, 0) \neq A_0^0 + A_1^0]$, but the initial conditions with respect to the average temperature and temperature of the gases are fulfilled exactly. This is explained by the circumstance that in deriving (28) we assumed the fulfillment of boundary condition (16) for a temperature distribution according to (19) at all instants, beginning with zero. This means that the relation

$$t_g(Z) = t(0, Z) + \left(1 + \frac{n}{\mathrm{Bi}}\right)[t(1, Z) - t(0, Z)], \quad (35)$$

always holds between the temperatures of the gases on the axis and surface of the heated body. Actually (35) is fulfilled only for $Z \geq 0.3$. If it is required to refine the solution for $Z < 0.3$ and a uniform initial distribution of temperatures, we can use the idea of the depth of penetration of a thermal wave [8]. In metallurgical heat engineering the main heating period, when (28)-(30) are valid, is of greatest interest.

Condition (35) is fulfilled when calculating the heating of bodies in a counterflow and location of point Z = 0 at the site of discharge of the metal, since the distribution of temperatures in the heated bodies at point Z = 0 is not arbitrary but is governed by the preceding process of heating during which relation (35) was observed. Therefore, if $m_{01} > 0$, $m_1 \geq 0$ (i.e., t_{gi} is also known), one of the following values should be assigned:

$$t_{av}(0) = A_0^0 + A_1^0 \frac{2\nu + 2}{2\nu + 2 + n}, \quad t(0, 0) = A_0^0, \quad t(1, 0) = A_0^0 + A_1^0. \quad (36)$$

Then we obtain two equations for determining A_0^0 and A_1^0: one of them is determined by (36) and the other by (35) written for Z = 0:

$$t_{gi} = A_0^0 + A_1^0 \left(1 + \frac{n}{\mathrm{Bi}}\right). \quad (37)$$

If $m_{01} \equiv 0$ and t(1, 0) and $\Delta t_m^0 = t(1, 0) - t(0, 0) = A_1^0$ are assigned according to the technological conditions (i.e., A_0^0 and A_1^0 are known), then t_{gi} and m_1 are determined by (37) and the second of relations (27). If we assign only m_1 and $t_{av}(0)$, or t(0, 0), or t(1, 0) is known, then t_{gi}, A_0^0, and A_1^0 are determined for system of Eqs. (36), (37) and the second of relations (27).

The exponent n can be selected either by comparison with the exact solution or experimentally. In the case of heating bodies in an environment with a constant temperature, and also in a parallel flow and counterflow when $w_g = m_0 = \text{const}$ and moderate values of Bi, we can take approximately $n \approx 2$ [10].

It follows from a comparison of approximate formulas (9), (11), (28), (29) with the exact solution of problem (1)-(4) obtained by Laplace transformation, which is shown in Fig. 1, that, beyond the limits of the initial period, the expressions given here, especially (28) and (29), approximate the exact solution sufficiently well. Since the values of the temperature of the gases are quite close to the exact also in the initial period, the temperature $t(\rho, X)$ in this period, generally speaking, can be refined with the use of the known solutions [9] for boundary conditions of the third kind and variable temperature of the medium. It is interesting to note that Eq. (10) gives the squares, taken with the opposite sign, of the approximate values of the eigenvalues, of which the first two are quite close to the exact (see Table 1).

We must assume that when $m_1 > 0$ there will be a satisfactory agreement of the exact and approximate solutions. Figure 2 presents the results of calculating the heating of a plate in a counterflow for $m_{01} = 0$, $m_1 = 0.25$, $Bi = 1.0$, $Bi_1 = 0.02$.

Expression (9), (11), (13), and (28)-(30) can be used for analyzing the effect of various factors on heating of bodies and for constructing the optimal regime.

NOTATION

t, t_g, r, x	are the current temperatures of the body and gases, radius, and distance from the entrance of the bodies into the furnace;
a, λ, α	are the coefficients of thermal diffusivity, thermal conductivity, and heat transfer;
R	is the radius of the cylinder, sphere, or the half-thickness of the plate;
K	is the coefficient of heat losses of the working space;
π	is the effective perimeter of the furnace;
v	is the linear velocity of the bodies in the furnace;
f	is the heating surface of the bodies located per unit length of the furnace;
w_g	is the water equivalent of the combustion products;
G	is the output of the furnace;
v_g	is the quantity of combustion products passing through a given section of the furnace in unit time;
c, c_g	are the heat capacities of the heated bodies and combustion products;
Q_n^r	is the heat of combustion of a unit of fuel;
L_g	is the quantity of combustion products formed upon burning a unit of fuel;
γ	is the specific weight of the material of the heated bodies;
ν	is the form factor [1];
$\nu_{pl} = 1/2$;	
$\nu_{cy} = 0$;	
$\nu_{sp} = 1/2$;	
$q_3^{av}(y)$	is the average chemical undercombustion of the combustion products;
t_{en}	is the temperature of the environment far from the furnace;
z	is the distance from the gas input into the furnace (for a parallel flow $z = x$, for a counterflow $z = L - x$);
L	is the length of the furnace occupied by metal;
b	is the width of the furnace occupied by metal;
s_1	is the distance between the axes of adjacent cylinders;
F	is the cross-sectional area of the furnace (shaft) filled with spheres;
f_{01}	is the ratio of the volume of voids to the total volume of the furnace (shaft);
t_g^0	is the temperature of the combustion products entering the furnace through the side burners;
$K_e = k_e vR^2/a$, $K_s = k_s vR^2/a$;	
q	is the specific heat flux absorbed by the heated body.

Subscripts

pl denotes the plate;
cy denotes the cylinder;
sp denotes the sphere;
$L_1 = aL/vR^2$.

LITERATURE CITED

1. E. M. Gol'dfarb, Heat Engineering of Metallurgical Processes [in Russian], Metallurgizdat, Moscow (1967).
2. G. P. Ivantsov and B. Ya. Lyubov, Dokl. Akad. Nauk SSSR, 85, No. 5, 993 (1952).
3. V. N. Timofeev, in: Scientific Transactions of VNIIMT [in Russian], No. 8, Metallurgizdat, Sverdlovsk (1962), p. 472.
4. Yu. G. Yaroshenko and F. R. Shklyar, in: Investigations of Thermal Conductivity [in Russian], Nauka i Tekhnika, Minsk (1967), p. 472.
5. M. Ya. Brovman and E. V. Surin, Inzh.-Fiz. Zh., 4, No. 2 (1961).
6. I. N. Bronshtein and K. A. Semendyaev, Mathematics Handbook [in Russian], Fizmatgiz, Moscow (1955).
7. M. K. Kleiner and N. Yu. Taits, Inzh.-Fiz. Zh., 7, No. 7 (1964).
8. T. V. Gudmen, in: Problems of Heat Transfer [in Russian], Atomizdat, Moscow (1967), p. 41.
9. A. V. Lykov, Theory of Thermal Conductivity [in Russian], GITTL, Moscow (1952).
10. A. V. Kavaderov and V. N. Kalugin, in: Transactions of VNIIMT, "Heating of Metal and Operation of Heating Furnaces" [in Russian], No. 6, Metallurgizdat, Sverdlovsk (1960), p. 3.